ISSN 2229-5518

COMPARATIVE ANALYSIS OF QUADRUPLE CONICAL TANK AND CYLINDRICAL TANK SYSTEM

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Abstract : Most of the industrial process are highly nonlinear and dynamic. Design of multivariable control system is of great demand in the process industry. System having more than one actuating input and then control output are consider as a multivariable system or MIMO system. Designing of quadruple tank level control system is most challenging. Depending upon the shape and size of the tank system behavior also different. Here we compare the quadruple conical tank and cylindrical tank process.

1 INTRODUCTION

In process industries, the control of quadruple tank system is most challenging due to nonlinear behavior. Control of quadruple tank system is major problem in industrial process**.** Quadruple tank system is used to analyses the nonlinear effect in a multivariable process. Thus helps in realizing the multi loop systems in industries. In quadruple tank system we need to control the two input and two output. The quadruple tank system is widely used in visualizing the dynamics interaction and nonlinear exhibits in the operation many process industries such as Chemical industries, Power plant, Fertilizer industries. Many industrial control problems have more number of manipulated variables. It is common for industrial processes to have significant uncertainties, strong interaction of minimum and non-minimum phase behavior. In this process there are two phase, minimum phase and non-minimum phase. The two phases can be obtained by changing valve controlling flow ratio γ_1 and $\gamma_2$between the upper and lower tanks.

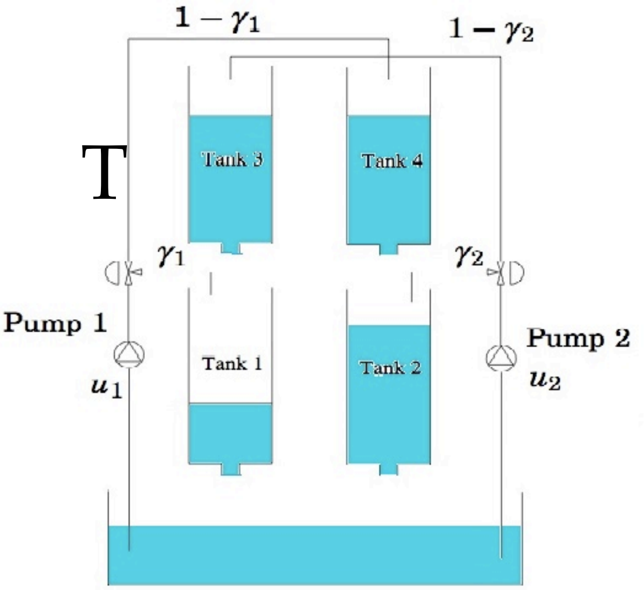
In this paper represents the system behavior of quadruple conical and cylindrical tank. These processes consider 4 interconnected tanks with minimum and non-minimum phase configuration.

2 QUADRUPLE TANK SYSTEM

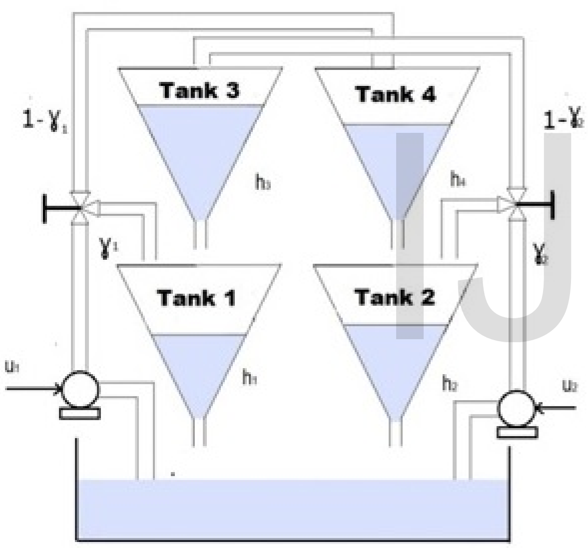
The quadruple tank system works in a multi input multi output system that could be used to analyze different control strategies. It is considered as a two double-tank process. It consists of four interacting tanks, two pumps and two valves. The two process inputs are the voltages v1 and v2 supplied to the two pumps. Tank 1 and tank 2 are placed below tank 3 and tank 4 to receive water flow by the action of gravity. Reservoir is present in the bottom to accumulate the outgoing water from tank 1 and tank 2 . Every tank has a valve which is fitted to its outlet. The action of pumps 1 and 2 is to suck water from the reservoir and deliver it to tanks which is based on the valve opening. Pump 1 delivers water to tank 2 and tank 3. Similarly the pump 2 delivers water to tank 1 and tank 4. From upper tanks, the lower tanks receive water due to gravitational force .The system aims at controlling the liquid levels in the lower tanks. The controlled outputs are the liquid levels in the lower tanks (h1, h2). The valve positions are γ_1 and γ_2. These valve positions give the ratio in which the output from the pump is divided between the upper and lower tanks. The flow rate can be monitored using the two rotameters and flow to the tanks can be adjusted by pump positions and. The valve position is fixed during the experiment and only the speed of pump is varied by changing the input voltage. The operation of quadruple tank system can be comprehended in two phase's namely minimum phase and non- minimum phase.

Minimum Phase: When the liquid enter the lower tanks is less than that of upper tanks then the system starts operating in minimum phase.

Non-Minimum phase: when fraction of liquid entering the upper tanks is less than that of lower tanks, then the system starts operating in non-minimum phase.



Quadruple Cylindrical Tank Process



Quadruple Conical Tank Process

3 MATHEMATICAL MODEL

Variables	**Description**	**Units**
H	Height of conical tank	cm
h_i	Height of liquid level	cm
h_1, h_2	Steady state height of liquid levels of tank 1&2.	cm
h3,h4	Steady state height of liquid levels of tank 1&2.	cm
Vi	Voltage of the pumps(i=1&2)	Volt
γ1 γ2	Flow distribution to lower and diagonal upper tank of Valve 1. Flow distribution to lower and diagonal upper tank of Valve 1.	
A	Cross sectional Area of tank	cm2
V	Volume of the conical tank	cm2
A	Cross sectional Area of outlet	cm2
Qi	Pump's flow	cm3/sec
G	Acceleration due to gravity	980 cm/sce2

K_1 and K_2 are Pump ConstantThe process inputs are V_1 and V_2. And the process outputs are h_1 & h2.

The output from level measurement devices

$$Y_1 = K_c h_1 \text{ and } Y_2 = K_c h2$$

Where ,K_c= Gain of sensor =1.

Flow from pumps to Tanks:-

	Tank 1	Tank 2	Tank 3	Tank 4
Pump 1	$\gamma_1 K_1 V_1$	-	-	$(1-\gamma_1) K_1 V_1$
Pump 2		$\gamma_2 K_2 V_2$	$(1-\gamma_2) K_2 V_2$	

For derivation mathematical model of quadruple tank, we set up basic equation that hold for each of the tanks and for the two pumps. They are put together to obtain the model of whole system.

Transfer function of quadruple cylindrical tank :-

Mass Balance equation:-

$V = Ah = q_i - q_{out}$

Where, V = Volume of tank

A = Cross sectional area of the tanks

h = Water level hight

q_i = Inlet flow, q_{out} = Outlet flow = $a\sqrt{(2gh}$

Pump Generated flow:-

$Q_{pump\,j} = K_p V_j$ K_p = Pumps flow constant.

V_j = Input voltage to the pump (j=1, 2)

Now,

The state equation:-

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}u_1$$
$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}u$$
$$\frac{dh_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}u_2$$
$$\frac{dh_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}u_1$$

State space representation of the system

$\frac{dx}{dt}$= Ax + Bu

y =Cx + Du

$\frac{dh}{dt}$ = f (h, v)

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$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}-\frac{1}{T_1} & 0 & \frac{1}{T_3} & 0\\ 0 & -\frac{1}{T_2} & 0 & \frac{1}{T_4}\\ 0 & 0 & -\frac{1}{T_3} & 0\\ 0 & 0 & 0 & -\frac{1}{T_4}\end{bmatrix} \cdot \begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} +$$

$$\dot{x} \quad \begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix} \cdot \begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}$$

The with the time constants T_i such that $\frac{1}{T}$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}-\frac{1}{T_1} & 0 & \frac{1}{T_3} & 0\\ 0 & -\frac{1}{T_2} & 0 & \frac{1}{T_4}\\ 0 & 0 & -\frac{1}{T_3} & 0\\ 0 & 0 & 0 & -\frac{1}{T_4}\end{bmatrix} \cdot \begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}\frac{\gamma_1 k_p}{a} & 0\\ 0 & \frac{\gamma_2 k_p}{a}\\ 0 & \frac{(1-\gamma_2)k_p}{a}\\ \frac{(1-\gamma_1)k_p}{a} & 0\end{bmatrix} \cdot \begin{bmatrix}u_1\\ u_2\end{bmatrix}$$

$$\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix} \cdot \begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}$$

with the time constants T_i such that $\frac{1}{T_i} = \frac{a\sqrt{2g}}{A} \cdot \frac{1}{2\sqrt{h_i}}$

Transfer Function Matrix:-

$$G(s) = C(sI-A)^{-1}B, \quad \begin{bmatrix}Y_1(s)\\ Y_2(s)\end{bmatrix} = G(s) \cdot \begin{bmatrix}U_1(s)\\ U_2(s)\end{bmatrix}$$

$$G(s) = \begin{bmatrix}\frac{T_1\gamma_1 k_1}{A(1+sT_1)} & \frac{T_1(1-\gamma_2)k_2}{A(1+sT_3)(1+sT_1)}\\ \frac{T_2(1-\gamma_1)k_1}{A(1+sT_4)(1+sT_2)} & \frac{T_2\gamma_2 k_2}{A(1+sT_2)}\end{bmatrix} =: \begin{bmatrix}g_{11}(s) & g_{12}(s)\\ g_{21}(s) & g_{22}(s)\end{bmatrix}$$

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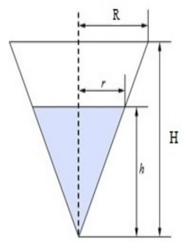
$$\frac{dh1}{dt} = \frac{3}{A1}\{a\sqrt{(2gh_3)}+\gamma_1 K_1 v_1 - a\sqrt{(2gh_1)}\}$$
$$\frac{dh2}{dt} = \frac{3}{A2}\{a\sqrt{(2gh_4)}+\gamma_2 K_2 v_2 - a\sqrt{(2gh_2)}\}$$
$$\frac{dh3}{dt} = \frac{3}{A3}\{(1-\gamma_2)K_2 v_2 - a\sqrt{(2gh_1)}\}$$
$$\frac{dh4}{dt} = \frac{3}{A4}\{(1-\gamma_1)K_1 v_1 - a\sqrt{(2gh_4)}\}$$
$$1/T_1=\frac{3\,a\sqrt{(2g)}}{A1\,(2\sqrt{h1})},\quad 1/T_1/=\frac{3\,a\sqrt{(2g)}}{A1\,(2\sqrt{h3})},$$
$$1/T_2=\frac{3\,a\sqrt{(2g)}}{A2\,(2\sqrt{h2})},\quad 1/T_2/=\frac{3\,a\sqrt{(2g)}}{A2\,(2\sqrt{h4})},$$
$$1/T_3=\frac{3\,a\sqrt{(2g)}}{A3\,(2\sqrt{h3})},\quad 1/T_4=\frac{3\,a\sqrt{(2g)}}{A4\,(2\sqrt{h4})}.$$



SYSTEM ANALYSIS

The rate of change of liquid height (h1, h2, h3, h4) and relates the difference between inlet flow and outlet flow.

Cylindrical Tank:

Common Parameter:-

H=Tank Height (40 cm)

R= Radius of the cylinder =7 cm

r=Radius of the outlet =1cm

The state space representation:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} (-1/T_1) & 0 & 1/T_1' & 0 \\ 0 & (-1/T_2) & 0 & 1/T_2' \\ 0 & 0 & 1/T_3 & 0 \\ 0 & 0 & 0 & 1/T_4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 3\gamma_1 K_1/A1 & 0 \\ 0 & 3\gamma_2 K_2/A2 \\ 0 & (1-\gamma_2)K_2/A2 \\ (1-\gamma_1)K_1/A2 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = \begin{pmatrix} K_c & 0 & 0 & 0 \\ 0 & K_c & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

Parameter	Minimum Phase	Non-minimum Phase
h_1, h_2, h_3, h_4	4cm,5cm,2cm,3cm	6cm,4cm,2cm,1.5cm
γ_1 , γ_2	0.7 0.8	0.43 0.30
K_1 , K_2	3.3 3.32	3.14 3.3
A	153.86 cm^2	153.86 cm^2
A	3.14 cm^2	3.14 cm^2
T_1 T_2 T_3 T_4	4.42,4.94,3.12,3.83	5.42,4.42,3.13,2.70

Transfer Function Matrix:

$$G(s) = C(sI-A)^{-1}B, \quad \begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = G(s)\cdot\begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}$$

$$G(S)=\begin{pmatrix} \dfrac{3\gamma_1 K_1}{A_1\,(S+(1/T_1))} & \dfrac{3(1-\gamma_2)K_2}{A_3 T_1'\,(S+(1/T_1))(S+(1/T_3))} \\ \dfrac{3(1-\gamma_1)K_1}{A_4\,T_2'(S+(1/T_2))(S+(1/T_4))} & \dfrac{3\gamma_2 K_2}{A_2(S+(1/T_2))} \end{pmatrix}$$

$$= \begin{pmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{pmatrix}$$

Transfer Function :-

$$G(S)=\begin{pmatrix} \dfrac{0.06678}{(1+4.42S)} & \dfrac{0.0213}{(1+3.12S)(1+4.42S)} \\ \dfrac{0.02862}{(1+4.94S)(1+3.83S)} & \dfrac{0.0852}{(1+4.94S)} \end{pmatrix}$$

ISSN 2229-5518

Minimum Phase:

Non-Minimum Phase

$$G(s) = \begin{pmatrix} \frac{0.047558}{(1+5.42S)} & \frac{0.06636}{(1+5.42S)(1+3.13S)} \\ \frac{0.063042}{(1+4.42S)(1+2.7S)} & \frac{0.02844}{(1+4.42S)} \end{pmatrix}$$

Conical Tank:

Common Parameter:-

H=Tank Height (40 cm) R= Outer radius of Conical tank.=12cm

r=Radius of the outlet =1cm & a=cross sectional area of the outlet = $3.14x1^2$=$3.14cm^2$

Parameter	Minimum Phase	Non-minimum Phase
h_1, h_2, h_3, h_4	20cm,18.69cm,14.83cm,16.984cm	21.399cm,18.69cm,14.83cm,13.48cm
γ_1 , γ_2	0.7 0.8	0.25 0.30
K_1 , K_2	3.32 3.3	3.3 3.15
A_1, A_2, A_3, A_4	$98.7167cm^{2,}$ $113.04cm^2$,$62.15.1cm^2$, $81.517\ cm^2$	129.407 cm^2,$98.7167cm^2$,$62.1519cm^2$ $51.3513cm^2$
a	$3.14\ cm^2$	$3.14\ cm^2$
$\frac{1}{T1},\frac{1}{T2},\frac{1}{T3},$	0.488,0.4127,0.8718,0.62104	0.34851,0.48884,0.87165,1.106557
$1/T_1^{/}$ $1/T_2^{/}$	0.5482 0.4478	0.418641 0.57561

$$G(S) = \begin{pmatrix} \frac{0.019125}{(S+0.34851)} & \frac{0.04455}{(S+0.34851)(S+0.87165)} \\ \frac{0.083227}{(S+0.4884)(S+1.106557)} & \frac{0.028718}{(S+0.4884)} \end{pmatrix}$$

Transfer Function :-

Minimum Phase

$$G(s) = \begin{pmatrix} \frac{0.0706}{(S+0.488)} & \frac{0.017429}{(S+0.488)(S+0.8718)} \\ \frac{0.01641}{(S+0.4127)(S+0.62104)} & \frac{0.070}{(S+0.4127)} \end{pmatrix}$$

Non-minimum Phase:

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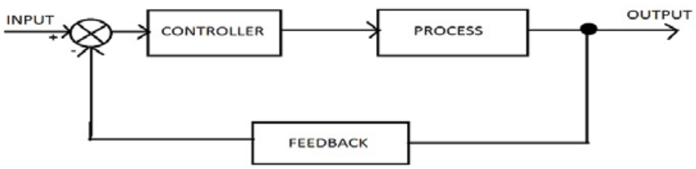
SIMULATION RESULT

Using the transfer function the following response for Quadruple tank for both minimum and non-minimum phase. The block Diagram of closed loop response shown in the figure-1.

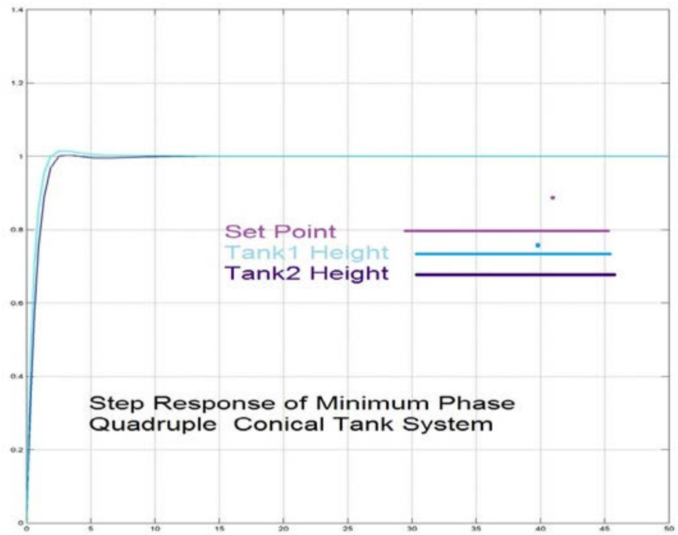


Comparison of response both Cylindrical Tank and Conical Tank:-

Minimum Phase:

Cylindrical Tank Best PI Controller Value :

Controller 1	Controller 1
P=15	P=26
I=4	I=4



Minimum Phase:

Conical Tank Best PI Controller Value :

Controller 1	Controller 1
P=28	P=18
I=9	I=9

Non Minimum Phase:

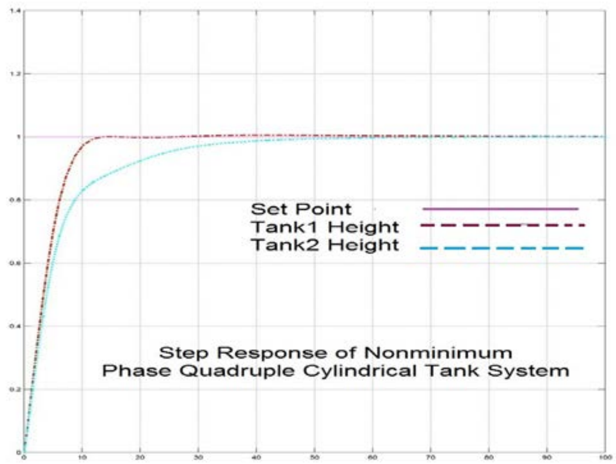
cylindrical Tank Best PI Controller Value :

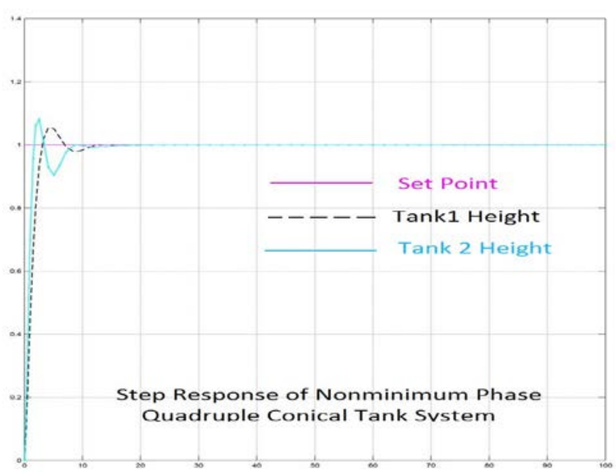
Controller 1	Controller 1
P=14	P=17
I=2	I=1.5

NonMinimum Phase:

Conical Tank Best PI Controller Value :

Controller 1	Controller 1
P=18	P=16
I=4.8	I=4.4

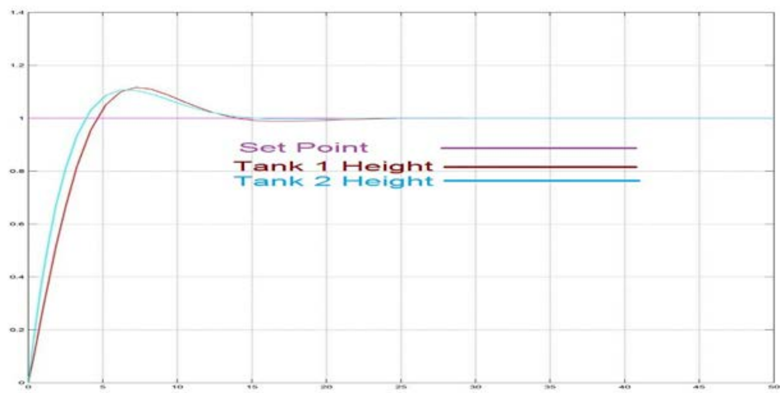




Now Change PI Value of Controller:-

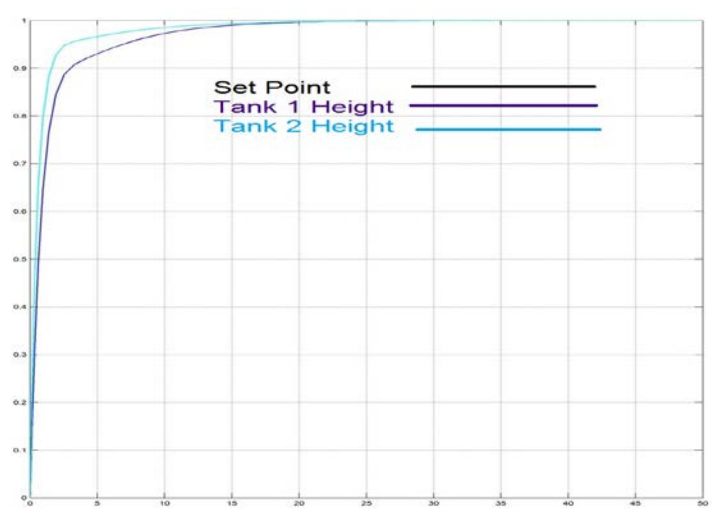
For Cylindrical Tank (Minimum Phase)

	Controller1	Controller2
	P=18	P=28
	I=9	I=9



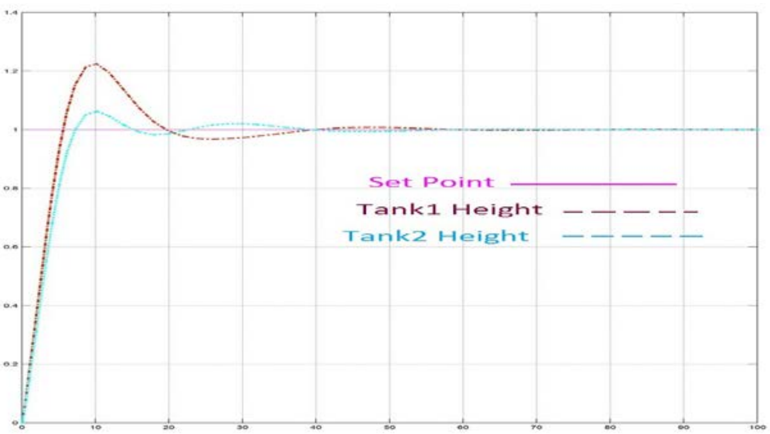
For Conicall Tank (Minimum Phase)

	Controller1	Controller2
	P=15	P=26
	I=4	I=4



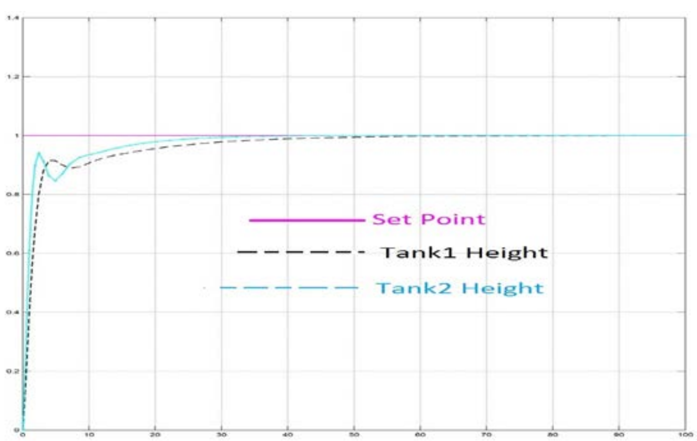
For Cylindrical Tank (NonMinimum Phase)

	Controller1	Controller2
	P=14	P=17
	I=2	I=1.5



For Conical Tank (NonMinimum Phase)

	Controller1	Controller2
	P=16	P=18
	I=4.8	I=4.4



CONCLUSION

For this simulation studies the initial condition is considered to be a steady state condition. In general it is not possible to tried an accurate model specially for a system which is non linear and so designing of the controller for desired output becomes difficult .The techniques which are used for modeling also becomes time consuming. Here we compare Mathematical model of the quadruple conical tank system and cylindrical system. The main difference of this two system is that in a conical tank each height of the liquid level cross sectional areas changes but in cylindrical tank cross sectional areas remains constant.

The Step Response of Quadruple Cylindrical tank and Quadruple Conical tank with PI controller is analyzed. Due to the more number of tank the order of system is increase and nonlinearity increase .It is observed that P I value of Quadruple Conical tank is more than the Quadruple Cylindrical tank because nonlinearity of conical tank is very high

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International Journal of Engineering and Technology (IJET

ISSN 2229-5518

Design of Decentralised PI Controller using Model Referance Adaptive Control for quadruple tank process

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